

Definability and Interpolation in Non-Classical Logics

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Algebraic approach to study of classical and non-classical logical calculi was developed and systematically presented by Helena Rasiowa in [1], [2]. It is very fruitful in investigation of non-classical logics because it makes possible to study large families of logics in an uniform way. In such research one can replace logics with suitable classes of algebras and apply powerful machinery of universal algebra.

This paper is devoted to interpolation and related properties of non-classical logical systems. Interpolation theorem proved by W.Craig in 1957 for the classical first order logic was a source of a lot of investigation devoted to interpolation problem in various logical theories. H.Rasiowa [3] proved interpolation theorem for many-valued predicate calculi. Interpolation is considered as desirable and "nice" property; also it has important practical applications in computer science. Interpolation is closely related to Beth's definability properties, which are implied by interpolation in many logical systems.

The original definition of the interpolation property admits various analogs which are equivalent in the classical logic but non-equivalent in other logics. The same is true for the Beth property. Here we consider several versions of the interpolation property and of the Beth definability property and present a picture of their inter-relations in different logics. In addition to the traditional forms of interpolation (Craig's interpolation and deductive interpolation) we take into consideration also their restricted variants.

In this paper we give an overview of results on interpolation and definability in propositional modal and positive logics, and also in extensions of Johansson's minimal logic. All these logics are strongly complete under algebraic semantics. It allows to combine syntactic methods with studying varieties of algebras and to find algebraic equivalents for interpolation and related properties. Moreover, we give exhaustive solution to interpolation and Beth's definability problems over the intuitionistic and some other logics.

One can find some of these results in our recent book [4] but the most of them still exist only in papers, and partially are not yet published.

The research is supported by INTAS, grant no. 04-77-7080.

References

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