

Remarks on A. Mostowski's philosophy of mathematics*

Roman Murawski

Adam Mickiewicz University

Faculty of Mathematics and Comp. Sci.

ul. Umultowska 87

61–614 Poznań, Poland

E-mail: rmur@amu.edu.pl

In the review of Mostowski's *Thirty Years of Foundational Studies* published in *Studia Logica* R. Suszko (1968) characterized Andrzej Mostowski as a “mathematician-logician, to whom the philosophical aspect of logic and the theory of the foundations of mathematics is not alien” (p. 169). The aim of the present paper is to analyze philosophical views of Mostowski expressed in his papers and books and to consider their influence on his (technical) studies and results in the domain of mathematical logic and the foundations of mathematics as well as vice versa.

Let us start our considerations by noticing that Mostowski was aware of the importance of philosophical questions. In the Introduction to the book *Logika matematyczna* [Mathematical Logic] (1948) Mostowski clearly stated that among difficulties that should be overcome when one is writing a book on the mathematical logic there is the fact that we cannot deprive logic (independently of the degree of it being formal) of a certain (at least subconscious) philosophical base. A conscious choice of such a base is difficult because we do not know with a certainty neither which of the many views being in conflict is the best nor which is at least a good one.

The same consciousness of the importance of philosophical questions one finds also in the Introduction to the monograph *Teoria mnogości* [Set Theory] (1952) written together with K. Kuratowski. The authors are convinced that the meaning and importance of set theory for the foundations of mathematics were demonstrated also in connection with the philosophy of mathematics. In their opinion set theory provides a *tool* for other parts of mathematics which are directly connected with applications.

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Mostowski came back to philosophical problems of set theory also later, in particular when he commented (in a series of papers — cf. 1964, 1967a, 1967b, 1968) Cohen's results on the independence of the axiom of choice and of the continuum hypothesis.

He considered also philosophical problems in connection with Gödel's incompleteness theorems (cf. 1946, 1957). And again, as it was in the case of set theory, he indicated only the philosophical problems connected with the discussed mathematical issues and showed the possible solutions but avoided any fixed and definite philosophical declarations. Moreover the philosophical comments were reduced to a minimum.

In Mostowski's opinion the main problem that should be solved in the philosophy of mathematics is the problem of the foundations of set theory as well as the problem of the genesis of mathematical concepts and the problem of laws governing the development of mathematics. Those problems have been considered by him in the paper (1955a, 1955b) — the only one in which he made definite philosophical declarations.

Analysis of Mostowski's philosophical remarks leads to the conclusion that he was aware of philosophical problems connected with mathematics and its foundations and of their importance and meaning. On the other hand he tried to avoid (with few exceptions) any definite philosophical declarations concentrating instead on strongly mathematical and technical side of issues. He was aware of the meaning of results obtained in the foundations of mathematics by mathematical methods for the philosophy of mathematics but simultaneously was convinced that those results cannot give definite solutions to problems of the philosophical nature. Therefore he rather presented various possible solutions instead of making any concrete declarations. Philosophical problems and possible solutions to them were discussed on the margin of proper metamathematical and foundational studies, in introductory remarks only and — what is very important — did not influenced the latter. He strongly avoided philosophical comments and remarks in technical papers. Philosophical perspective on the one hand and metamathematical and foundational one on the other were strictly separated by him. In this way any possible philosophical presuppositions did not restrict methods available and admissible in mathematical and metamathematical investigations used by him. On the other hand one may suppose that some of Mostowski's researches and achievements could be inspired by philosophical ideas, could have grown up from philosophical considerations (e.g., independence of definitions of finiteness, classification of logical systems or Kleene-Mostowski hierarchy, construction of models with automorphisms). But he never made any remark on that — therefore there is no base to suggest any hypotheses.

What were the reasons and sources of the described attitude of Mostowski? They can be seen in the attitude and ideology of Lvov-Warsaw Philosophical School and of Warsaw School of Logic — Mostowski belonged to the latter generation of this school (cf.

Murawski, 2004). In fact Polish logicians and mathematicians — being convinced of the importance of philosophical problems and knowing quite well the current philosophical trends — treated logic and mathematics as autonomous disciplines independent of philosophical reflection on them, independent of any philosophical presuppositions. Therefore they sharply separated mathematical and logical research practice and philosophical discussions concerning logic and mathematics. Philosophical views and opinions were treated as “private” matter that should not influence the mathematical and metamathematical investigations. On the contrary, in the latter all correct methods could and should be used. This “methodological Platonism” enabled Polish logicians and mathematicians to work in various areas without being preoccupied by philosophical dogmas. In controversial cases, as for example in the case of the axiom of choice in set theory, their attitude can be characterized as neutral — without making any philosophical declarations they simply considered and studied various mathematical consequences of both accepting and rejecting the controversial principles and investigated their role in mathematics.

The attitude of Mostowski towards the philosophy of mathematics shown in the above analysis of his works fits quite well into the described scheme.

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