RYSZARD WÓJCICKI
Institute of Philosophy and Sociology, Polish Academy of Sciences
in collaboration with
JAN ZYGMUNT
Wrocław University

POLISH LOGIC OF THE POSTWAR PERIOD

1. Introductory Remarks. During the 10th Congress of Logic, Methodology and Philosophy of Science (Florence, August 19-25, 1995), I took part in a panel discussion on the situation of logic in Eastern Europe during the time of Soviet domination. This essay, originally written to celebrate the 50th anniversary of the Polish Academy of Sciences[i], is primarily based on the paper I presented on that occasion.[ii] The list of people with whom I consulted while preparing first the Florence paper and then the present one is rather long.[iii] I appreciate the assistance of all of them. My special thanks are due to Wojciech Buszkowski, Andrzej Grzegorczyk, Witold Marciszewski, Wiktor Marek, Roman Murawski, Jerzy Tiuryn and Jan Zygmunt, who in addition to offering various suggestions, remarks and criticism provided me with brief overviews of selected areas of logical investigations carried out by Polish logicians.

The postwar Polish logic is too rich and too diversified for one person to be able to present it in an adequate manner, and I would not have been able to complete this paper without these people's kind assistance. Yet, it goes without saying that the final responsibility for this paper is mine. I tried as far as I was able to evaluate critically all pieces of information I was offered and occasionally revise them. In this manner, the Polish version of this survey was prepared in cooperation with Jan Zygmunt, who was also the author of the initial draft of its English version. The latter was accessible on the Studia Logica home page for quite some time. Eventually, I rewrote its various parts taking into account suggestions offered to me by its readers. Of those, I particularly

appreciate the remarks and comments offered by Z. Adamowicz, J.M. Dunn, W. Marek, R.Murawski, Z. Pawlak, J. van Benthem, and H. Wansing. I also express my gratitude to Tom Brunty who took the effort to polish this translation.

This paper was, in its initial form, addressed to a rather large audience, and though this time its expected readers are logicians, its "popular' style has been preserved. Thus, in particular, various rather "loose" definitions meant to explicate various technical notions to the reader whose knowledge of logic is limited were neither removed nor replaced by definitions thought to be more accurate.

2. Definition. The term „logic" is rather ambiguous, yet the question of how it should be understood is crucial for delimiting the scope of this paper. Thus, even though no answer to this question is likely to gain approval from all whose opinion on the matter should be considered, I do not think I should avoid undertaking it.

"Logic" (in the basic meaning of the term) is the formal theory of reasoning. Thus, it is a theory whose chief concern is to state conditions that an argument should satisfy in order to be valid. One method of approaching this issue consists of reducing the notion of the validity of an argument to that of the validity of a „rule of inference". The idea here is that an argument is valid if and only if its conclusion has been derived from premises by using valid rules of inference.

The logical analysis of reasoning is carried out under the assumption that both the premises and the conclusion are sentences rather than someone's beliefs or suppositions. The sentence is a syntactic concept and quantity and thus it should be defined in terms of vocabulary and the syntactical rules that govern the language examined. On the other hand, the notion of validity is related to that of truth and thus is a semantic one. Consequently, its purely syntactical definition is not available.

As is rather clear from the remarks above, logical investigations, both in their syntactic and semantic versions, are bound to be strongly related to and often to overlap linguistic ones. Even though logicians limit their linguistic analyses to languages deprived of numerous peculiarities characteristic of "natural" languages (i.e., to languages one might call "ideal" or "idealized"), those languages are by no means „artificial", as is often claimed. The relationship between idealized languages as the objects of logical analyses and natural languages is much the same as that between theoretical models of phenomena and the phenomena they represent, e.g. between a system of mass points whose behavior is described by Newtonian Mechanics and the system of corresponding real, three-dimensional physical objects.

If sentences examined by logicians were treated as someone's beliefs or suppositions, the logical theory of reasoning would be both relevant to and dependent upon results of investigations into human mental capacities. From the point of view of cognitive psychology, reasoning is not merely a sequence of operations on sentences (as the logicians maintain), but is a mental activity of a human being. If logic is not a part of psychology, as was maintained until the beginning of the 20th century, it is not because it treats notions like reasoning, belief or supposition as merely informal and thus dispensable, but because its laws are not experientially grounded. Logic is a formal science and, as all formal sciences, notably mathematics, it is concerned with some „arbitrarily chosen" set of assumptions that define the objects (or rather the system they form) to be investigated. The domains of logical investigations are known as „logical systems". The fact that a logical system is determined by „arbitrarily chosen" assumptions does not mean that the selection of such assumptions is not governed by restrictions. Whenever a new logical system is proposed, one expects that it will be useful in dealing with either theoretical or practical problems of considerable significance.

Most often a system of logic is defined as a set of sentences (more broadly, well-formed formulas) that are „logically true", i.e. true regardless of how one understands the "non-logical terms" they contain. Since the chief task

of logic is to state conditions for valid reasoning, besides defining the notion of logical truth, the definition of a logical system should instruct the user as to how the concept of logically valid inference might be reduced to that of logical truth. An alternative way of defining logic consists of defining the notion of a logically valid inference directly, rather than in terms of logical truth. Thus there are two alternative ways to define a logical system. One might respectively call them "sentential" (where the logical truth of a sentence is of primary substance) and "inferential" (where the key notion is that of a valid inference).

Seeking to comply with different logical intuitions, logicians arrive at different systems of logic, or different „logics" for short. The system known as classical logic is of special significance for it provides a formal basis for all standard mathematics.

3. The Outskirts of Logic. Not without good reason, mathematicians consider logic as one of the foundations of mathematics. Two other key elements of mathematics are set theory and recursion theory. The tendency to identify logic with mathematical foundations is most natural. If one adopts such a perspective, then in addition to logic in the narrow sense (which I sought to characterize in the previous section), set theory and recursion theory, one must perforce also treat many other forms of mathematical investigation as parts of logic. For instance, one cannot help but treat investigations into the foundations of arithmetic as part of logic, for their main subject is the logical analysis of various possible formalizations of arithmetic. For the same reason, one cannot resist classifying investigations into foundations of geometry, foundations of algebra, etc, as part of logic Similarly, category theory, universal algebra, and a relatively young discipline called complexity theory might also be regarded as branches of modern logic.[iv]

The logical foundations of computer science define another important area of investigation strongly related to logic. The relevance of logic for computer science owes to several key attributes. First, the structure of computing machines is based on the laws of logic. Second, logic provides the tools that are

necessary for the design and analysis of programming languages. Thirdly, „cognitive engineering", or the design of data bases and expert systems, fruitfully uses logical tools. The language of predicate calculus combined with the laws of logical deduction provide the general framework for defining computer languages. Therefore, along with foundations of mathematics, the foundations of computer science are also rightfully regarded as part of logic in a general sense of the word.

The foundations of mathematical linguistics and the formal foundations of communication theory are two more branches of science which employ the ideas and techniques of logic. In these cases it is extremely difficult to draw a precise border between logic and issues that do not belong to it. Finally, there is another discipline related to logic in many essential ways; namely, cognitive science, including the theory of artificial intelligence. The relations between logic and cognitive science may be readily discernible , but they cannot, however, be easily characterized. One reason for this is that, in spite of its various considerable achievements, cognitive science remains a conglomerate of concepts and ideas that split into often dramatically different areas of investigations.

In this essay I will make every effort to maintain a reasonable balance between the narrow (presented in Section 2) and the wide (presented in this Section) conceptions of logic. This will not be an easy task, and I only hope that both those who prefer to define logic in a narrow manner and those who opt for as broad a definition as possible may forgive my departures from the specific idea of logic they consider to be the right one.

4. The Interwar Period. The achievements of Polish logicians in the years 1920 – 1939 gained worldwide recognition. The term „Polish School of Logic" became popular within the international community of logicians. It is noteworthy that during this time the achievements of Polish logicians overlapped in a very substantial way those of another formation known as the „Lvov-Warsaw School of Philosophy".[v]

Seen from today's perspective, the activity of the Polish School of Logic resulted in achievements whose significance can hardly be overestimated. It was during this time when Kazimierz Ajdukiewicz formulated his conception of categorial grammar (inspired by some ideas of developed by Tadeusz Kotarbiński), Jan Łukasiewicz developed his many-valued logic, and Adolf Lindenbaum and Alfred Tarski invented the method of algebra of language known today as the method of Lindenbaum algebra. This was also the period when Tarski published his fundamental papers on the conception of truth and deductive systems (following Kotarbiński's reism, the term coined by Kotarbinski for his radical nominalism) and Stanisław Leśniewski attempted to form a nominalistic version of the set theory strong enough to base mathematics on it. Leon Chwistek put forward the idea of a simplified theory of types, Janina Hosiasson-Lindenbaum examined the methodological and logical aspects of probability theory, Jan Łukasiewicz and Alfred Tarski developed an algebraic treatment of logical matrices, and Stanisław Jaśkowski (independent of the German logician Gerhard Gentzen) defined the system of „natural deduction" (a system of rules of inference that make the „formalized" inference similar to the „natural" one). This list is remarkable in that it recounts a vast array of spectacular achievements, but perhaps its most notable feature lies in the fact that it is far from complete.

Within the Polish School of Logic logical investigations were carried out both in philosophical and in mathematical departments. For instance, while Leśniewski and Tarski were members of the faculty of the mathematical department of Warsaw University, Ajdukiewicz (Lvov University), Kotarbiński, and Łukasiewicz (Warsaw University) represented the „philosophical wing" of logic. Thus, the cooperation between „mathematicians" and „philosophers" was systematic and close.[vi]

5. World War II. The Second World War was a disaster for both Polish logic and Polish science as a whole, for three obvious reasons.First, some of the logicians, including those who made substantial contributions to the development

of their discipline, did not survive. Priest Jan Salamucha, historian of logic and a close collaborator of Jan Łukasiewicz and Jacek Bocheński, was killed during the Warsaw Uprising in 1944. Because of their Jewish origin Adolf Lindenbaum, his wife Janina Hosiasson-Lindenbaum, Mojżesz Presburger, and Mordechaj Wajsberg (the author of significant results concerning many-valued logic) were murdered by the Nazis. Being abroad when the war started, Tarski avoided this tragic fate.

Second, the war destroyed the whole structure of cooperation among and between scientific institutions. The Jan Kazimierz University of Lvov and the Stefan Batory University of Vilnus ceased to exist. So did numerous research teams. The continuity of teaching, publishing, and organizational work was broken. The manuscripts of many papers being prepared for publication were destroyed. But still, during the war, the illegal „underground universities" were formed and acted on a large scale, offering among others courses of logic. Among its teachers and students were Kazimierz Ajdukiewicz, Janina and Tadeusz Kotarbiński, Andrzej Mostowski, Jan Salamucha, Zygmunt Zawirski, Henryk Hiż, Zbigniew Czerwiński, Andrzej Grzegorczyk, Jerzy Pelc, Roman Suszko, Klemens Szaniawski, and many others.

Finally, when the war was over and Poland was captured by the Soviet Union, many Polish logicians who were lucky enough to leave their country earlier decided to stay abroad. Thus, for instance, Jacek M. Bocheński settled in Switzerland, Czesław Lejewski in England, Jan Łukasiewicz in Ireland, and Henryk Hiż, Bolesław Sobociński, and Alfred Tarski in the USA. Andrzej Ehrenfeucht and Jerzy Mycielski were able to emigrate later.

6. Early Postwar Period – logic in philosophy departments. Although everyone was aware that both political and social life would undergo dramatic changes in postwar Poland, the end of the war brought new hopes. Despite tremendous losses, Polish logic did not cease to exist and soon began to experience a rebirth both in philosophical and mathematical university departments.

Tadeusz Kotarbiński and his wife Janina started their academic activity in Łódź, which, since Warsaw had been completely destroyed during the war and had not yet beenrebuilt, was the main scientific center of the country (the Łódź University was founded in 1945). Among the many young people grouped around them were Jerzy Pelc, Marian Przełęcki, and Klemens Szaniawski. At about this time Kazimierz Ajdukiewicz became the Rector of Poznań University. Besides his rector duties, he gathered a team of people who either used logic in their investigations or treated logic as the main subject of their scientific pursuits. Among others, Marcin Czerwiński, Jerzy Giedymin, Seweryna Łuszczewska-Romahnowa, and Roman Suszko worked under his leadership. Maria Kokoszyńska-Lutman, a student of the eminent Tarski who became recognized for her analyses of Tarski's notion of truth, settled in Wrocław. Henryk Mehlberg settled there along with her, after which he eventually succeeded in emigrating to the USA. In Cracow there was a group of logicians led by Izydora Dąmbska and Roman Ingarden. Tadeusz Czeżowski moved from Vilnius to Toruń, where one of his best known students was Leon Gumański.

Except for Roman Ingarden (one of the most prominent Polish philosophers of the 20th century) all the philosophers and logicians whom I mentioned as promoters of newly formed postwar research groups were outstanding representatives of the Lvov-Warsaw School. Besides Kazimierz Twardowski (cf. footnote v), Ajdukiewicz and Kotarbiński were perhaps the most prominent members of that formation. Not all of them (one might also mention here Izydora Dąmbska) were logicians, even in the widest sense of the term. All of them, however, appreciated the significance of logic and were ready to act in support of its development. Zygmunt Zawirski (professor at the mathematics department of the Jagiellonian University in Cracow) was an outstanding example of one displaying such an attitude. Under his supervision, Stanisław Jaśkowski (1945), Andrzej Mostowski (1945), and Jerzy Słupecki (1947) completed their habilitation theses, and R. Suszko (1945) received his master degree.

7. Early Postwar Period – logic in mathematics departments. During the early postwar period it was customary to avoid the term "logic" as either the name or part of the name given to sections of the mathematics departments in which logicians were grouped. The preferred term was "foundations of mathematics." This terminology was a form of camouflage. Even though the ideologists of the communist party considered no branch of science to be ideologically neutral, logic (viewed as part of philosophy and thus part of a discipline supposed to be the chief tool of "ideological offensive") was the subject of their special attention. At different times and in different countries, it was demanded of logicians (with varying degrees of vigor) to suppress formal logic and supplant it with the so-called „dialectical logic" implicit in the writings of Hegel and the „classics" of Marxism.

In Wrocław logic was primarily developed within two groups. One of them was led by professor Czesław Ryll-Nardzewski and the other by professor Jerzy Słupecki. Ryll-Nardzewski, at that time the author of several seminal papers on logic, grouped several talented people around himself, among them L. Pacholski, B. Węglorz, A. Wojciechowska. Among those who started their academic careers under Słupecki's supervision were Witold A. Pogorzelski, Ludwik Borkowski, Bogusław Iwanuś, Tadeusz Prucnal, and also Jerzy Loś, one of the most brilliant logicians of the postwar period (and the author of the widely known Ultraproduct Theorem).

Logic was also being developed by scholars elsewhere in Poland. Notable among them is Stanisław Jaśkowski, who began his work in the mathematics department of the Nicolas Copernicus University in Toruń in 1945. Among others his collaborators there included Jerzy Kotas and August Pieczkowski.

Polish postwar logic owes a great deal to professor Andrzej Mostowski, who pursued his academic career in Warsaw. He was such a prominent figure that his achievements deserve to be presented in a separate section.

8. Andrzej Mostowski. There are several reasons for considering A. Mostowski an exceptional scholar. First, he was an unquestionable scientific authority. Some of his results concerning the foundations of mathematics were of breakthrough significance. His contribution to set theory was immense. He was one of those who, together with A. Tarski, developed the theory of decidability (a theory whose main task is to settle whether, for a given theory, there is an algorithm that enables one to distinguish sentences that are its theorems from the remaining ones). He suggested an algebraic interpretation of the quantifiers and initiated the research of the so-called generalized quantifiers.[vii] He also made a substantial contribution to model theory.

Second, he was a scientist of very well established and international reputation. Many eminent researchers in logic and foundations of mathematics visited Poland solely for the purpose of contacting him.

Third, he had a unique skill to look at logic as a discipline whose methods are mathematical and yet whose issues are philosophical in origin. His textbook *Logika Matematyczna* („Mathematical Logic") published in 1948 combines in an exemplary way an exposition of the "technicalities" of mathematical logic with a fairly large discussion of the philosophical aspects of its problems. Two of his other works are of a similar nature: *The Present State of Investigations on the Foundations of Mathematics*[viii] and *Thirty Years of Foundational Studies*.[ix] The latter will be discussed in section 14.

Among his students and close collaborators were numerous stellar representatives of the postwar movement in logic and foundations of mathematics: Zofia Adamowicz, Andrzej Ehrenfeucht, Andrzej Grzegorczyk, W. Guzicki, Wiktor Marek, Helene Rasiowa, Roman Sikorski, Paweł Zbierski, and many others.[x] At the department of mathematics of Warsaw University Mostowski was the Head of the Section of Algebra and the Section of the Foundations of Mathematics. He also was the director of the Institute of Mathematics at the Polish Academy of Sciences and a member of the Polish Academy of Sciences. His personal prestige and position translated into prestige and position

for logic as a whole, andthis surely contributed to the further development of the discipline.

9. Alfred Tarski. Speaking at the Florence Congress (see the introduction) I stated the following: „One might say, not being entirely wrong, that for a long period of time the chief Polish seminar on logic was held in Berkeley, California, in the residence of Alfred Tarski, who kept very close cooperation with his friends and colleagues from Poland, Andrzej Mostowski in particular." From the 1940's onward Berkeley was visited by Wanda Szmielew, Andrzej Mostowski, Andrzej Ehrenfeucht, Jerzy Łoś, Jerzy Mycielski, Lesław Szczerba, Leszek Pacholski, and others, who took part in various research programs initiated by Tarski.[xi]

Tarski was undoubtedly one of the most outstanding logicians of the 20th century. He also influenced immensely the development of research in semiotics and philosophy. Philosophical aspects of his theory of truth (Tarski was not only fully aware of them, but also put them forward in his papers published in philosophical periodicals) are even at present the subject of vivid controversies and analyses.[xii] Intuitions associated with the notion of truth formed a background for his concept of the consequence operation, as well as (cf. section 15) the model theory.

The research center on foundations of mathematics created by Tarski in Berkeley was one of the most influential in the world. Despite many difficulties (among them getting permission to travel abroad), Polish logicians were able to keep in touch with him and his students (who are among the most outstanding American logicianstoday), the importance of which for postwar Polish logic is hard to overestimate.[xiii]

10. The First Congress of Polish Science. In four previous sections I adduced some evidence that Polish logic continued to develop despite severe war losses. But even though this growth was achieved, growing ideological pressures and various decisions of a political or administrative nature posed very real

threats to its continued development. Necessarily, the climate created by these factors had direct consequences for the continued development of logic.

Today hardly anybody is aware that the discussive logic put forward by Jaśkowski in 1948 was an attempt to form a logical system that admits controversies and contradictions in discussions. Jaśkowski did not care to call those controversies „dialectical" or to use the word „dialectical" on any occasion in his papers. Notably, he also never cared to quote any of the so-called „classics" of Marxism (which was still possible in Poland, though in Russia that would have been an act of desperate courage).

Jaśkowski's papers (as well as a few more published by Maria Kokoszyńska-Lutman, Ludwik. S. Rogowski, and Tadeusz Kubiński) were essentially attempts to argue, albeit implicitly, that there is a chance for cooperation between formal logic and dialectical logic; the former might serve as a useful tool for clarifying and analyzing the rather enigmatic ideas of the latter. This attempt was bound to fail, since dialectical logic was supposed to be a part of so-called „Marxist dialectic" and not reducible to any system of logic.

The postulate of submitting science to ideology, along with some institutional changes, was introduced at the First Congress of Polish Science in 1953. Logic was not the main aim of the „ideological offensive" there; rather, the Lvov-Warsaw School was. Party ideologists were committed to begin the process of full eradication of any politically incorrect philosophy (i.e., those different from „diamat" –dialectical materialism, and „hismat" – historical materialism) from academic life. Poland could not be allowed to remain an oasis, free from the rules being obeyed in other countries of the Soviet block, most notably the Soviet Union.

The public criticism of the Lvov-Warsaw School was directed against its most distinguished representatives: Kazimierz Ajdukiewicz, Tadeusz Kotarbiński, Maria Ossowska, and Stanislaw Ossowski. It is noteworthy , however, that this campaign did not result in the eventual expulsion of the „bourgeois" philosophers from academic life. Instead, measures were instituted to prevent them from teaching philosophy, humanities or social sciences. Some of the

philosophy professors were employed as the chairs of logic. Others received jobs in the Polish Academy of Sciences.

11. Ajdukiewicz's Initiative for Expanding Logical Investigations. It is interesting to note that although the future of logic was becoming more and morein jeopardy , one of the Party's chief ideologists – Adam Schaff, whose aim as chief critic of Ajdukiewicz in the 1953 ideological debate was to eradicate bourgeois philosophy from universities – actually backed Ajdukiewicz's effort to organize the Section of Logic of the Polish Academy of Sciences. That section eventually became a part of the Institute of Philosophy and Sociology of the Academy.

Along with forming the section of Logic, Ajdukiewicz founded *Studia Logica* (SL), an academic journal intended to provide a forum for both mathematical and philosophical logicians.[xiv] The original editorial board consisted of the following scholars: Kazimierz Ajdukiewicz (editor in chief), Leszek Kołakowski, Tadeusz Kotarbiński, Andrzej Mostowski, and Roman Suszko (secretary). Unfortunately the presence of L. Kołakowski in this assemblage does not mean that this eminent philosopher of world-wide contemporary recognition began his career as a logician. The reason for his presence at the editorial board was rather peculiar, though certainly characteristic of that time. At least one person on the Editorial Board had to be a Party member, and no other person invited to join the Editorial Board satisfied this condition.

12. Fifty Years of *Studia Logica*. As was intended by Ajdukiewicz, *Studia Logica* played from its very beginning the role of a bridge linking mathematical and philosophical logic. Shortly after its inception the periodical started to appear regularly, with an extended editorial board, and its papers came to be published more and more often in foreign languages, which in turn attracted authors from abroad. Just as Ajdukiewicz had intended, logicians from mathematical departments came fairly often to choose Studia Logica as a forum to present the results of their investigations alongside of their publications in

mathematical journals. In this respect, Andrzej Mostowski had a significant effect in his work on the editorial board. After K. Ajdukiewicz's death in 1963, Jerzy Słupecki (the Department of Mathematics of Wrocław Univeristy) became the editor in chief. In 1971 Zdzisław Pawlak and then in 1978 Helena Rasiowa (both true luminaries in the field of mathematical logic) joined the editorial board. In 1987 Rasiowa went on to become one of the managing editors of the journal.

In 1976 *Studia Logica* underwent an important transformation. Thanks to the efforts of Ryszard Wójcicki (at that time the Editor-in-Chief) and other members of the editorial board (notably Stanisław Surma, Klemens Szaniawski and Jan Zygmunt) the international editorial board was established.[xv] Thus began the process of transforming this periodical into a truly international publication, which was not easy, especially due to the fact that any and all international contacts were supposed to be cleared by the State Security Police.[xvi]

Since 1953 over 150 issues of SL have been published. From the time it became an international periodical in 1976,English has been its sole language of publication, and its editorial board is of broad international constituency. Moreover, the Ossolineum Publishing Company, the original publisher of the journal, succeeded in getting North-Holland to become a co-publisher. A few years later North-Holland was replaced by Kluwer Academic Publishers, and in 1991 Kluwer became the sole publisher of *Studia Logica*.[xvii] From 1991, Ryszard Wójcicki, who at the end of the 1980's had stepped down as Editor-in-Chief to be replaced by Jan Zygmunt, resumed the responsibility of heading the journal (assisted by Jacek Malinowski). Since 2000, *Studia Logica* has been published in three volumes a year. In 1995, with the publication of SL's companion series, *Trends in Logic*, the Studia Logica Library was started.

13. Publications on Logic. *Studia Logica* has not been the only Polish periodical open for publishing papers on logic. Various logical papers, notably some directly related to foundations of mathematics, have been published in *Fundamenta Mathematicae* (FM), an international journal established in 1920.

FM has been a vehicle for a large number of papers written by logicians from all over the world. The scope of this journal has been rather large from its very beginning, and it has covered foundational studies into mathematics of all kinds.

In 1973 *Reports on Mathematical Logic* was founded by Stanisław Surma. The profile of *Reports* has been much the same as *Studia Logica*. A newsletter called *Bulletin of the Section of Logic of the Institute of Philosophy and Sociology of the Polish Academy of Sciences*[xviii] has been active in promoting both research and international cooperation in logic. In 1993 Jerzy Perzanowski started *Logic and Logical Philosophy*, a periodical fully devoted to philosophical logic.

Since 1974, *Fundamenta Informaticae* (a journal initiated by Zdzislaw Pawlak) has been published under the editorship of Helena Rasiowa and then Andrzej Skowron, who assumed the editorship after Rasiowa's death. This journal covers applications of logic to computer science and is published by IOS Press.

This list would not be complete without mentioning the series *Biblioteka Myśli Semiotycznej* (*The Library of Semiotics Ideas*) initiated by Jerzy Pelc. It is hard to overestimate its importance, not only for semiotics but also for logic. The 46 volumes of this series give the reader some understanding of the richness and variety of the research that has occurred in those areas where logic meets linguistics.[xix]

14. The Foundations of Mathematics. Even the barest attempt to present only the most important achievements of the research into the foundations of mathematics would require a separate paper. Hence, such a presentation certainly cannot be provided within this section.[xx] A highly competent and informative survey of this area was offered, however, by A. Mostowski`s in his study, *Thirty years of Foundational Studies* (cf . footnote 7). This work covers the developments of 1930 – 1964, and its list of references consists of 244

items. This reference table is truly outstanding, and some of its most remarkable citations are as follows:

A. Ehrenfeucht is mentioned both as the author of a paper on the methods of game theory applied to the problem of decidability of the first order theories and as co-author (together with A. Mostowski) of a paper on model theory. This reference mentions six papers by A. Grzegorczyk, one of them written jointly with A. Mostowski and C. Ryll-Nardzewski. Grzegorczyk's papers, with one exception, concern the problems of decidability and computability, while the joint paper is related to some foundational problems of arithmetic. The decidability theory is also the subject of the paper by A. Janiczak. J. Łoś is credited with a list of six papers, mainly on model theory. One of them, written with C. Ryll-Nardzewski, explores the theory of representation of Boolean algebras and the problem of Stone theorem's equivalents. The paper by Łoś and Suszko is concerned with the operation of summing models. E. Marczewski's paper on abstract algebra is mentioned along with a large work by S. Mazur on computational analysis. Mostowski mentions four of his own works, and lists H. Rasiowa as the author of several papers on algebraic representation of various non-classical logics. Algebraic methods are the subject of a cited work by H. Rasiowa and R. Sikorski, who also authored the monograph *The Mathematics of Metamathematics* (PWN, 1963, position [173] on the list). Mostowski cites two papers by C. Ryll-Nardzewski;one of them presents a characterization of categorical theories, and in the other the axiom of induction is examined and it is proved that Peano arithmetic is not finitely axiomatizable. In the paper by R. Sikorski the notion of a metric space is applied to the analysis of intuitionistic logic. W. Szmielew proves in her paper the famous theorem stating the decidability of the elementary theory of abelian groups. Tarski's name appears nine times as an author and several times as a co-author,while the monograph *Undecidable Theories* (North-Holland) is jointly attributed to A. Tarski, A. Mostowski, and R. M. Robinson. This book set an important direction in the search for general methods of proving the undecidability of formalized theories in general and various mathematical theories in particular.

If one were to create an even more extensive list of the names, discoveries, and achievements that have been important since 1964, one would certainly need to mention the following. Z. Adamowicz (whose findings concerned both so-called „weak arithmetic" and arithmetic with open induction), K.R. Apt (second order arithmetic and foundations of computer science), A. Ehrenfeucht, A. Grzegorczyk and W. Guzicki (several works concerning the notion of forcing), M. Krynicki (writings related to the notion of generalized quantifiers), A. Mostowski (reflexivity of Peano arithmetic), R. Murawski (expandability of models of Peano arithmetic to models of second order arithmetic), S. Krajewski (nonstandard satisfaction classes), W. Marek, and M. Srebrny (their investigations were concerned with higher-order arithmetic and set theory, in particular with relations between Zermelo-Frankel theory and Kelley-Morse class theory), H. Kotlarski (automorphisms of nonstandard models of Peano arithmetic), J. Mycielski (infinite combinatorics, universal algebra, and the analysis of Hugo Steinhaus's axiom of determinacy), Z. Ratajczyk, C. Ryll-Nardzewski, L. Szczerba (foundations of geometry, the notion of interpretability of theories formulated in various languages), T. Traczyk (Hilbert spaces, quantuum logic), P. Zbierski (descriptive set theory), and Z. Vetulani (foundations of second and higher-order arithmetic and artificial intelligence).

The following works also should be mentioned. In the 1970's, A. Grzegorczyk published the textbook *Zarys Logiki Matematycznej*, whose third edition (Polish Scientific Publishers, 1973) was translated into English as *An Outline of Mathematical Logic* (Kluwer 1974). Z. Adamowicz and P. Zbierski wrote *Logika Matematyczna* (PWN, 1991), translated as *Logic of Mathematics*: *A Modern Course of Classical Logic* (John Willey & Sons, 1997).

In retrospect, it is now obvious that even after the period covered in his landmark tome (i.e., since 1964), A. Mostowski [xxi] played the leading role in the development of the foundations of mathematics and its important branch – model theory (cf. section 15) – while A. Tarski played a somewhat less direct though still prominent role in this regard.

15. Model Theory. This theory has a distinguished position within the foundations of mathematics. Initiated in the 1940's-50'sby the work of L. Henkin, A. Robinson, and A. Tarski, it forms an important part of mathematical logic. It deals with the relations between language in its formalized version and its „models", i.e. the structures that may become the objects to which the language expressions refer (if they are properly interpreted).

As was pointed out in the previous section, many important results in model theory were obtained by the Poles. In the 1960's and 1970's model theory was developed by L. Pacholski and B. Węglorz, later by H. Kotlarski, and quite recently important and widely recognized results were obtained by Ludomir Newelski. An especially significant advance in this field (because of its numerous applications) was the development of Łoś's ultraproduct theorem, which was based on some of Skolem's ideas of the 1930's. The Boolean models method of H. Rasiowa and R. Sikorski[xxiii] was another important milestone, since it enabled logicians to extend the field of model theory to non-classical logics.

16. Algebraic Logics. The history of this branch of logic (overlapping universal algebra to some extent) goes back to Lindenbaum-Tarski's idea of the algebra of language. For the sake of brevity, suffice it to state that algebraic logics deal with the relations between the laws of logic and theorems that characterize algebraic operations.[xxiv]

The basic tools for examining the relations between algebra and logic (or more precisely, between algebraic structures and particular systems of logic) were established by Alfred Tarski along with his collaborators and followers. The monograph by H. Rasiowa and R. Sikorski, *The Mathematics of Metamathematics*, PWN, 1963 (cf. 20), an outstanding source of information in this area, was a source of inspiration for authors all over the world to carry out research in algebraic logic. In 1974 H. Rasiowa published her monograph, *An Algebraic Approach to Non-Classical Logic*[xxv] in which she extended her earlier results to the large class of so-called non-implicative logics. This class includes modal logics among others.

Besides Rasiowa and Sikorski, algebraic logic was also the research subject of Cecylia Rauszer, who was interested in logics with „constructive falsehood". Many important and difficult results were obtained by A. Wroński and his group of logicians (Section of Logic, Philosophy Department of the Jagiellonian University in Cracow). Their research, carried out partly in cooperation with Japanese logicians, made important contributions to the theory of pseudo-Boolean algebras, BCK algebras, equivalential algebras and logical systems related to these classes of algebras. In Toruń, algebraic methods were developed by J. Kotas and his group.

The issues underlying algebraic logics are closely related to those of logical matrices (cf. section 19).

17. Foundations of Computer Science. The research in this branch of logic, the importance of which is still growing, was initiated by H. Rasiowa and her group (Grażyna Mirkowska, Ewa Orłowska, Cecylia Rauszer, Andrzej Salwicki, Andrzej Skowron, Jerzy Tiuryn, Anita Wasilewska, to name but a few). The activity of these people was oriented toward developing applied logic such as algorithmic logic, logic of programming (correctness of programs), logic of information (data bases), and logic of knowledge (expert systems). An important role in the development of the foundations of computer science was played by Zdzisław Pawlak („rough sets" theory, Pawlak machines), and also by Andrzej Blikle, Beata Konikowska, Józef Winkowski.

Research into the mathematical foundations of computer science was initiated in Poland by H. Rasiowa and Zdzisław Pawlak. Since the 1990's they have been undertaken in the Department of Mathematics, Mechanics, and Computer Science at Warsaw University by the group headed by Jerzy Tiuryn. This group consists of Damian Niwiński, Jerzy Tyszkiewicz, Paweł Urzyczyn, and Igor Walukiewicz, and deals with numerous aspects of modern computer science: lambda calculus, functional programming, type theory, modal logics, and theory of finite models.

18. Theory of Consequence. The first postwar papers with reference to Tarski's consequence theory were published by J. Łoś, J. Słupecki and W. A. Pogorzelski. One of the key papers on this subject was „The Algebraic Treatment of the Methodology of Elementary Deductive Systems"[xxvi] by J. Łoś. The research in this area was carried out also by A. Grzegorczyk and R. Suszko among others. It was Suszko who first developed the idea of „abstract logic".[xxvii]

There are two important aspects of the theory of consequence. Firstis the methodological aspect. Tarski developed the consequence theory as a theory of „deductive systems". This notion is a formal equivalent to that of „theory" or even „a system of beliefs". Logical analysis of a deductive system has numerous and significant methodological implications. Second, the notions of consequence let us see the limitations of the propositional conception of the system of logic and bring into prominence the inferential concept of those systems.[xxviii]

Many profound realizations concerning the so-called structural completeness of logical calculi (which was also examined abroad) were made by W.A. Pogorzelski (who introduced thisidea) and his students, including T. Prucnal and P. Wojtylak among others.

The area of consequence theory also includes the research concerned with so-called non-monotonic reasoning. This kind of research was carried out by Witold Łukasiewicz, Wiktor Marek and Mirosław Truszczyński.

19. Logical Matrices. The concept of a logical matrix, understood as a set of „logical values" (e.g. truth and falsehood in the simplest case) that can be taken by propositions, appeared in the beginning of the 20th century. J. Łukasiewicz introduced many-valued matrices while defining many-valued logics. A. Lindenbaum proved that each propositional logic (cf. section 2), i.e. set of tautologies, has an adequate logical matrix that characterizes it in a unique way. This result, along with one of the papers by A. Tarski and J. Łukasiewicz, showed

that logical matrices can be used as certain generalizations of algebraic methods (the latter were discussed in section 16).

J. Łoś and R. Suszko tried to find a matrix representation of consequence operation. Their result, however, required some corrections (given by R. Wójcicki).[xxix] These efforts (the former is of vital importance) gave a new direction to a very general, technically difficult, and philosophically interesting field of research.

This problem was my special concern in 1970 when I initiated a separate research project. I started to examine the representations of consequence operations together with J. Czelakowski, W. Dziobiak, J. Hawranek, M. Tokarz, T. Prucnal, R. Suszko, J. Zygmunt, A. Wroński, and P. Wojtylak. Some contributions to this project were made by logicians from abroad (e.g. W. Rautenberg), although this subject is regarded as typically „Polish".[xxx] This project was continued and developed later on by J. Czelakowski and W. Dziobiak as well as by Spanish and American logicians, especially J.M. Font, W.J. Block, and D. Pigozzi.[xxxi]

20. Type Logics and Categorial Grammars. These areas belong to the foundations of mathematical linguistics. They were explored in Poznań by the logicians grouped around Wojciech Buszkowski. The term „type logics" refers to the set of operations that are used while creating complex language expressions out of simple ones. „Type logics" belong to the family of substructural logics.

From the historical point of view this kind of research was initiated by Ajdukiewicz and Bar-Hillel, and refers to the calculus of Lambek (1958). Buszkowski`s research (as well as that of M. Kandulski and W. Zielonka) produced a thorough understanding of some of the fundamental problems of mathematical linguistics and its applications. These researchers explored the algebraic and computational properties of various kinds of languages, especially the so-called „tree-like languages" generated by categorial grammars, the algorithms for finding minimal categorial grammars, and the relations between

categorial grammars and Chomsky`s generative grammars.[xxxii] Contemporarily, research in logic and linguistics is also carried out in the Poznań center by T. Batóg and J. Pogonowski.

21. Suppositional Logic. In 1934 S. Jaśkowski published an article, *On the Rules of Suppositions in Formal Logic* (published as the first issue of the *Studia Logica* series[xxxiii]). Ironically, Jaśkowski's and Gentzen's conceptions of natural deduction were discovered independently of each other and yet were published almost simultaneously.[xxxiv] A suppositional system of logic was the basis for the computer program „Mizar" designed by Andrzej Trybulec (Warsaw Univerity's branch in Białystok, currently an independent university). Its function is well expressed by the title of one of the research projects coordinated by W. Marciszewski, „Systems of Logic and Algorithms for Computer Testing of the Correctness of Proofs".[xxxv]

22. Others. A very important institution, organized under the auspices of the Polish Academy of Sciences, is the Conference on the History of Logic. It was initially chaired by T. Czeżowski, J. Słupecki, and S. Surma, and since the 1980's it has been organized exclusively by A. Wroński, Section of Logic of the Jagiellonian University in Cracow. Substantial contributions to the subject of the history of logic were made by W. Marciszewski, R. Murawski, S. Surma, J. Woleński, and J. Zygmunt among others.

The creation of logical systems was not a Polish speciality. Besides the discussive logic of Jaśkowski (which was mentioned in section 10) there are two exceptions to this, however: The „non-Fregean logic" of R. Suszko – a system of logic inspired by some intuitions taken from *Tractatus* ... of L. Wittgenstein, and the system *Grz* (for Grzegorczyk, its author) – one of the most widely studied systems of modal logic.

Polish logicians (J. Słupecki, H. Rasiowa, T. Traczyk, G. Malinowski, W.A. Pogorzelski, T. Prucnal, M. Tokarz, K. Trzęsicki, and R. Wójcicki, not to mention the great precursors of the subject – J. Łukasiewicz, A. Tarski, and M.

Wajsberg) contributed tremendously to the research in many-valued logics.[xxxvi] Polish logicians also made important strides in the fields of intuitionistic logic (H. Rasiowa, R. Sikorski, P. Wojtylak, A. Wroński), modal logic (A. Grzegorczyk, E. Orłowska, J. Perzanowski, H. Rasiowa, C. Rauszer, J. Hawranek, and T. Skura), and relevance logic (W. Dziobiak, M. Tokarz, K. Swiridowicz).

Quite a large number of papers were concerned with deontic logic (J. Gregorowicz, L. Gumański, J. Kalinowski, T. Kubiński, W. Suchoń, K. Świrydowicz, J. Woleński, Z. Ziemba, Z. Ziembiński)and casual logic (H. Greniewski, A. Pieczkowski, K. Trzęsicki, and M. Urchs[xxxvii]). The latter were mostly based on some ideas initiated by S. Jaśkowski.

Polish authors have also dealt with the logic of questions (erotetic logic). This body of work was initiated by Ajdukiewicz and substantially developed by T. Kubiński, L. Koj, and A. Wiśniewski.

Scholarship conjointly embracing logic and theory of communication is being performed by M. Tokarz.

---

[i] The Polish version *Logika polska okresu powojennego, proba rzutu oka wstecz* of this paper was published in *Nauka* 4(2002), 157 – 175.

[ii] „The Postwar Panorama of Logic in Poland", in: *Logic and Scientific Methods*, eds. M.L. Dalla Chiara et al., Kluwer 1997, pp. 597-608.

[iii] Various suggestions regarding the earlier „Florence" version of this survey were offered by: Janusz Czelakowski, Andrzej Grzegorczyk, Jacek Malinowski, Marcin Mostowski, Roman Murawski, Ewa Orłowska, Witold A. Pogorzelski, Kazimierz Świrydowicz, Max Urchs, Jan Woleński, Andrzej Wójcik, Jan Zygmunt. While preparing this version of the survey I received assistance from: Zofia Adamowicz, Wojciech Buszkowski, Janusz Czelakowski, Witold Marciszewski, Wiktor Marek, Roman Murawski, Jan Mycielski, Mieczysław Omyła, Jerzy Pogonowski, Jerzy Tiuryn, Anita Wasilewski, Andrzej Wiśniewski, Jan Woleński, and Jan Zygmunt.

[iv] One might think that by extending the notion of logic to its limits, logicians behave like „logical imperialists" who try to invade other branches of mathematics. This is not so. Logic and its methods are both the source of inspiration and the basic research tool for mathematics. Applying the term "logic" to the foundations of mathematics enables the experts on foundational issues to establish their scientific identity. It also enables them to see their highly varied field of research as a whole that differs from the rest of mathematics.

[v] It was formed by a group of philosophers, logicians, sociologists, and other scientists who upheld the tradition initiated by the seminars and writings of Kazimierz Twardowski, an eminent psychologist and philosopher from Lvov University. Precise analysis and lucid argument were the virtues that Twardowski considered indispensable both in scientific inquiry and philosophical analyses. Logic was regarded as the basic tool for meeting this requirement. It is not odd then that the Lvov-Warsaw School attracted logicians while at the same time logicians and their works essentially influenced the School. An impressive monograph on the School was

written by Jan Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, Kluwer 1989. A brief and informative article on Polish logic of the interwar period can be found in *The Routledge Encyclopaedia of Philosophy*, vol 7, Routledge, London and New York, 1998, pp. 498-500, „Polish Logic" by J. Zygmunt.

[vi] Alfred Tarski – undoubtedly the most significant person in Polish logic during the interwar period, and one of the greatest logicians of the 20th century – dedicated the collection of his papers *Logic*, *Semantics, Metamathematics*, *papers from 1923 to 1938* (Clarendon Press, Oxford, 1956) to "my teacher Tadeusz Kotarbiński".
[vii] This subject was examined in many Polish and foreign centers. In Poland it was examined by: A. Krawczyk, M. Krynicki, L. Szczerba, W. Szmielew, M. Zawadowski, and others. Its computational aspects were analysed by A. Pawlak, H. Rasiowa, and E. Orłowska.
[viii] A. Mostowski (in collaboration with A. Grzegorczyk, S. Jaśkowski, J. Łoś, S. Mazur, H. Rasiowa, and R. Sikorski), „The Present State of Investigations on the Foundations of Mathematics", *Dissertationes Mathematicae* 9 (1955), pp. 1-48.
[ix] A. Mostowski, „Thirty Years of Foundational Studies; Lectures on the Development of Mathematical Logic and the Study of the Foundations of Mathematics in 1930-1964", *Acta Philosophica Fennica* 17 (1965), 1-180.
[x] The group of close collaborators of A. Mostowski included Janusz Onyszkiewicz, Stanisław Krajewski, and Konrad Bieliński. These names are well known to all who witnessed the democratic changes in Poland (J. Onyszkiewicz was the Defense Secretary in two cabinets, S. Krajewski remains one of the most prominent members of the Jewish community in Poland, and K. Bieliński was one of the leaders in the underground Solidarity movement). Of course there are other logicians (e.g. Jan Waszkiewicz) and scientists close to logic (e.g. Klemens Szaniawski, Rector Electus of Warsaw University) who were deeply engaged in the political movement that sought to promote democratic changes.
[xi] In a letter I received as part of correspondence concerning this paper, J. Mycielski wrote: „Since there was a close cooperation (exchange of papers and ideas) between Mostowski's and Tarski's schools, one can say that there was just one Berkeley-Warsaw school and it is impossible to discuss one without discussing the other."

[xii] Not only philosophical ones. Tarski's concept was the subject of numerous formal analyses and generalizations. An interesting survey of this topic can be found in S. Krajewski's essay „Prawda" in: *Logika Formalna. Zarys Encyklopedyczny z Zastosowaniami do informatyki i lingwistyki*, edited by W. Marciszewski, PWN 1987, pp. 144-156.
[xiii] Tarski's contribution to the development of logic is discussed in J. Zygmunt's „*Alfred Tarski*" in: *Polska Filozofia Powojenna*, vol. II, edited by W. Mackiewicz, Agencja Wydawnicza Witmark, Warszawa, 2001, pp. 342-375.
[xiv] The note „From the Editor" found in the first issue of *Studia Logica* reads that the journal will publish papers devoted to all areas of logic, including formal logic, mathematical logic, inductive logic, theory of definition and of classification, etc., and that SL invites especially works on the history of Polish logic.
[xv] Its membership included: N. D. Belnap, Jr and J.M. Dunn (USA), B. I. Dahn (DDR), L. Esakia (USSR, Georgia), D. Follesdal (Norway), R. Gilles (Canada), J. Hintikka (Finland), L. Maksimowa and V. A. Smirnow (USSR, Russia), R. Routley (Australia), I. Ruzsa (Hungary), P.Weingartner (Austria), and P. M. Williams (UK).

[xvi] This „party vigilance" was not unjustified. Since the scientific periodicals were not censored, there was a danger that some of the papers might have been written by „enemies of socialism". *Studia Logica* committed this kind of crime by publishing a review of A. A. Zinoviev's (one of the leading Soviet dissidents) book Logical Physics (SL 35, 1976). It was a book free from ideological issues, but still the members of the Soviet Academy of Sciences protested.

[xvii] According to the contract, the periodical remains one of the publications of the Institute of Philosophy and Sociology of the Polish Academy of Sciences, and the main Polish libraries receive it at reduced prices.
[xviii] The aim of this newsletter (founded by the section of logic of Inst. Phil. Soc. Pol. Ac. Sc. in 1973) was to extend international cooperation and indirectly promoting *Studia Logica*. Since 1991 it has been published by Łódź University and edited by Grzegorz Malinowski.
[xix] *Studia Semiotyczne* (founded by J. Pelc and published by Polskie Towarzystwo Semiotyczne-) has also played an important role in this area.

[xx] It requires a competence to which the author of this survey cannot aspire, nor probably can the workers in this discipline, because of its size and scope. On the other hand it would be odd not to mention the foundations of mathematics. The results of this discipline are strongly related to those of logic.
[xxi] The work of Andrzej Mostowski is discussed in five papers written by: A. Grzegorczyk, W. Guzicki, W. Marek, L. Pacholski, C. Rauszer, and P. Zbierski in: A. Mostowski, *Foundational Studies: Selected Works*, vol I, PWN, Warszawa, North-Holland, Amsterdam 1979.
The monograph which summarizes his metamathematical research in set theory is: A. Mostowski, *Constructible Sets with Applications*, PWN, Warszawa, North-Holland, Amsterdam, 1969.
[xxii] The work of Andrzej Mostowski is discussed in five papers written by: A. Grzegorczyk, W. Guzicki, W. Marek, L. Pacholski, C. Rauszer, P. Zbierski in: A. Mostowski, *Foundational Studies: Selected Works*, vol I, PWN, Warszawa, North-Holland, Amsterdam 1979.
The monograph which summarizes his metamathematical research in set theory is: A. Mostowski, *Constructible Sets with Applications*, PWN, Warszawa, North-Holland, Amsterdam, 1969.
[xxiii] H. Rasiowa and R. Sikorski, *The Mathematics of Metamathematics*, PWN, Warszawa, 1963 (3rd edition, 1970).
[xxiv] If by laws of logic one means the laws of classical logic, then they can be characterized by the laws of Boolean algebra. Non-classical logics can be represented by „non-Boolean" algebras(e.g. intuitionistic logic is characterized by pseudo-Boolean algebras).
[xxv] H. Rasiowa, *An Algebraic Approach to Non-Classical Logic*, PWN, Warszawa, North-Holland, Amsterdam, 1974.
[xxvi] *Studia Logica* 2, 1955, pp. 151-212.
[xxvii] This notion was also explored by Spanish logicians.
[xxviii] These two conceptions were discussed in section 2. The extent to which they differ from each other can be illustrated by the following. The system of 3-valued Łukasiewicz logic in its inferential meaning has two „non-trivial" extensions, while the same system in its sentential meaning has only one extension. This result (obtained by R. Wójcicki) was later generalized (G. Malinowski, W. Dziobiak). It makes clearer some of the „paradoxical" results obtained earlier by H. Hiż and Rasiowa, which were concerned with the sentential meaning of the logical system. The distinction between logic understood as a set of tautologies and a logical consequence enables us to show (R. Wójcicki) that we cannot define classical logic by the use of the constants of intuitionsitic logic in the inferential case (which is possible in the sentential case).
[xxix] The papers addressed here are: J. Łoś, R. Suszko, „Remarks on Sentential Logic", *Indagationes Mathematicae* 20, 1958, pp. 177-183, and R. Wójcicki „Some Remarks on the Consequence Operation in Sentential Logics", *Fundamenta Mathematicae* 68, 1970, 269-279.
[xxx] The results mentioned here as well as those of other logicians (W. Blok, H. Hiż, J. Łoś, D. Pigozzi, H. Rasiowa, W.A. Pogorzelski, and others) are discussed in: R. Wójcicki, *Theory of Logical Calculi; Basic Theory of Consequence Operations*, Kluwer, 1988. Some of these results were applied to the theory of automatic theorem proving (cf. Z. Stachniak, *Resolution Proof System, An Algebraic Theory*, Kluwer, 1996).
[xxxi] This research is discussed in the monograph: J. Czelakowski, *Protoalgebraic Logics*, Studia Logica Library, Kluwer, 2001.
[xxxii] A thorough discussion of their proceeds with a comparison to the proceeds of other centers results can be found in: W. Buszkowski „Mathematical Linguistics and Proof Theory", *Hand-*

book of Logic and Language (collective work edited by J. van Benthem and A. ter Meulen, Elsevier and MIT Press).

[xxxiii] S. Jaśkowski *On the Rules of Suppositions in Formal Logic*, Studia Logica. Wydawnictwo poświęcone logice i jej historji, nr 1, Seminarium Filozoficzne Wydz. Matematyczno-Przyrodniczego Uniwersytetu Warszawskiego, Warszawa, 1934. This series was to be published under the editorship of J. Łukasiewicz. Unfortunately only one volume was published. Postwar Studia Logica (cf. section 12) referred to it but did not comprise its continuation.

[xxxiv] In order to make S. Jaśkowski's system better known J. Słupecki and L. Borkowski elaborated a version of it (and published it in the book *Elementy Logiki i Teorii Mnogości*, PWN, 1963; translated as *Elements of Mathematical Logic and Set Theory*, Pergamon Press 1967) and presented the possibility of applying it to mathematical reasoning.

[xxxv] This project is supported by international grants (from EU and NATO, e.g. Ph. D. scholarships in Germany and Japan) and its main aim is to complete an online encyclopedia of mathematics (comprising a collection of computer verified proofs of theorems). There is also a quarterly: *Formalized Mathematics – A Computer Assisted Approach* published there, edited by R. Matuszewski.

[xxxvi] A good introduction to this subject is a book by G. Malinowski, *Many-Valued Logics*, Clarendon Press, Oxford 1993.

[xxxvii] M. Urchs is mentioned here because of his strong connections with Polish logic. He came to Poland as a young man from the DDR and began to study in Toruń.