A Further Generalization of Mostowskian Quantifier

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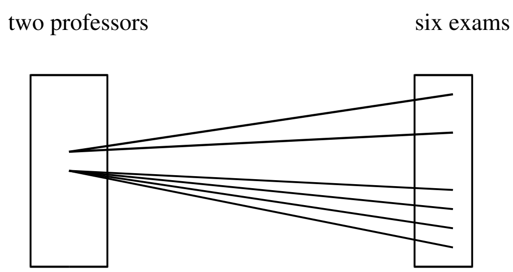
Abstract

This research aims at contributing to the problem of translating natural (ethnic) language into the framework of formal logic in a structure-preserving way.[1] Using logic as knowledge representation formalism has some well known merits but also presents some well identified problems. Some of them are connected with quantifiers. Introduction of Mostowskian generalized quantifier interpreted as a family of sets (Mostowski 1957) helped solving some of them, in particular those connected with undefinable NL-quantifiers. Still, problems of representing some natural readings for sentences with more then one quantifier remained unsolved. Let us consider one of them. Irena Bellert and others (cf. (Bellert, 1989)) identified four possible readings of the following sentence with NL quantifiers *two* and *six*:

(S) *Two professors marked six exams*

One of these four readings presupposes referring to exactly 2 professors and exactly 6 exams without specifying the quantity of exams marked by any of the two professors. The reading that makes trouble may be graphically represented as follows:



Reading 1

For this reading none of the two NL quantifiers is in the scope of another. Therefore, the direct application of the classical Mostowskian quantifier is useless for capturing this reading. The solution we

[1] The results presented here were first published at the FLAIRS 2005 Conference (Clearwater Beach, Florida) (cf. (Vetulani 2005).

propose consists in further generalization of the Mostowskian quantifier. Instead of considering the multi-quantifier sentence structure compositionally (as in classical approach) we consider the whole configuration of NL-quantifiers as a complex generalized quantifier and we propose its semantics (of Mostowskian type).

Within this approach any simple NL sentence involving several quantifiers is considered as a predicate-argument structure and a configuration of quantifiers corresponding to the argument positions. We propose the following notation for quantified formulas: $(Q_1x_1,Q_2x_2,...,Q_nx_n)P(x_1,x_2,...,x_n)$. The form $(Q_1x_1,Q_2x_2,...,Q_nx_n)$ is called *nonclassical quantifier*. For example the sentence (S) is represented by the formula:

(Two professors, Six exams) marked(professors, exams)

Contrary to the classical truth definition, the semantics of this complex quantifier will be defined for *particular readings* depending on the *context*. By *context* we mean *this factor which differentiates between readings*. The nature of this factor is (usually) pragmatic.

In the following formulas, the context (or reading) will be represented by the parameter C. As in case of Mostowskian truth condition, the interpretation $\|(Q_1x_1,Q_2x_2,...,Q_nx_n)\|_C$ will be some family of subsets of the universe and more precisely a family of subsets of the cartesian product $|U_1|\times|U_2|\times...\times|U_n|$ of domains corresponding to arguments, i.e.

$$\|(Q_1x_1,Q_2x_2,...,Q_nx_n)\|_C \subset P(|U_1|\times|U_2|\times...\times|U_n|)$$

The definition is as follows:

$$\models_C (Q_1x_1,...,Q_nx_n)\ P(x_1,...,x_n) \text{ iff } \{(x_1,...,x_n): \models_C P(x_1,...,x_n),\ x_i\in |U_i| \text{ for } i=1,...,n\}\in \|(Q_1x_1,...,Q_nx_n)\|_C$$

Now, what remains to do is to define the interpretations $\|Q\|_C$ for each configuration of quantifiers Q and each context (reading) C. Let us remark that the formula $(Q_1x_1,Q_2x_2,...,Q_nx_n)P(x_1,x_2,...,x_n)$, makes abstraction of the surface linear ordering of quantifiers in the sentence and also of the scope dependencies between quantifiers reflected in the traditional notation. This means that the predicate logic formulas $(\forall x(\exists yP(x,y))$ and $(\exists y(\forall xP(x,y))$ are both represented by one and the same formula $(\forall x,\exists y)P(x,y)$ and the scope relationship between $\forall$ and $\exists$ is to be encoded by the context/reading parameter.

Let us notice that in many free word order languages (e.g. Slavonic languages, Latin) the surface succession of quantifiers do not necessarily correspond to the scope relations (marked in some other way, e.g. lexically or inflectionally). Consistently, we conjecture that it is natural to consider information about these relations as a part of the formal context parameter.

The question concerning the nature of the context/reading parameter is of fundamental importance for both theory and applications. In her study of linguistically admissible readings of co-occurring quantifiers Irena Bellert (Bellert, 1989; Vetulani 1987) provides an interesting characterization of readings in terms of the linguistic features of absoluteness and distributiveness of quantifiers occurring in the sentence. E.g. the value *plus of distributiveness* of a quantifier with respect to a co-occurring quantifier indicates that the relation expressed by the predicate is distributed among all members of

some reference class of this co-occurring quantifier (or *the* reference class if this co-occurring quantifier appears to be +absolute). *Minus of distributiveness* indicates that the relation is distributed among *some,* not necessarily all, members of the reference classes of the corresponding quantified phrase. Plus/minus absoluteness informs whether the quantified phrase has one (plus) or more then one (minus) reference classes. For example Reading 1 of the sentence (S) is characterized by the following attribution of feature values :
"two professors":
  +absolute
  -distributive with respect to "six exams"
"six exams":
  +absolute
  -distributive with respect to "two professors".

The main observation made by Bellert (1989) is that not all of the possible configurations of feature values are linguistically admissible. The rules were proposed in application to pairwise co-occurring quantifiers in order to determine and characterize the linguistically admissible readings of quantifiers.[2]

What we claim (Vetulani 2005) is that configurations of feature values admitted by rules may be considered as candidates for context/reading parameters to be used in the definition of Mostowskian semantics for the *nonclassical quantifier* defined above. In particular, we claim that these configurations C of feature values will contain information necessary and sufficient to define the $\|Q\|_C$ classes.

References

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[2] The following three basic rules were proposed by Bellert (1989):
1. (Absoluteness) If Q_i is –absolute then there is an +absolute co-occuring quantifier Q_k such that Q_k is + distributive with respect to Q_i and Q_i is -distributive with respect to Q_k.
2. (Weak-symmetry) If Q_k is +absolute and Q_i is +distributive with respect to Q_k,, then Q_k is +distributive with respect to Q_i.
3. (Transitivity) If Q_i is -distributive with respect to Q_k and Q_k is -distributive with respect to Q_m, then Q_i is -distributive with respect to Q_m.